Graph Matching and Link Analysis for Dynamic Planning and Execution

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Abstract

This paper discusses an innovative approach to military campaign strategy; planning; resource allocation, and scheduling based on link analysis, graph matching and mixed linear integer programming. Graph matching explores purely topological aspects of planning and scheduling where the processes can be viewed as a graph. Link analysis explores relations among large numbers of similar or different types of objects. Mixed linear integer programming solves problems containing both integer variable constraints, like aircraft numbers, as well as linear (rational) variable constraints like risk and probability of destruction or success.

ATO Link (Air Tasking Order Link Analysis) combines these software technologies to more optimally use resources throughout all phases of the air campaign planning and execution process. These three mature, matrix-based and computationally well-behaved technologies (link analysis, graph matching, and mixed integer programming) have the potential to scale to extremely large plans, while scheduling limited resources more optimally.

The ATO Link prototype is based on an effects-based operations (EBO) approach for the aerospace planning domain, but we feel is applicable to all types of planning. This effort has the potential to significantly accelerate the military campaign planning and execution process while exploring more courses of action and maintaining plan rationale. The goal is more proactive, dynamic planning and execution to achieve full spectrum dominance in military operations.

Introduction

"On the timetable we'd promised the President, that would mean a seven-thousand-mile shift of five and one third divisions, or 120,000 troops, in four months. …The science of logistics had come so far since WWII … ***In theory all I had to do was push a button.*** … But, there was a big problem. Since we'd been in the middle of revising Central Command's battle plan when the crisis broke [Iraq invading Kuwait 2 August 1990 - Desert Storm], we hadn't entered the data into the computer banks—a painstaking process that under normal circumstances takes a full year. …I had never dealt with anything so complex, nor had to make so many decisions so

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quickly, in my life." pp 359-362. General H. Norman Schwarzkopf, *The Autobiography, It Doesn't Take a Hero.* General Schwarzkopf's "push one button" theory identifies a common misconception that joint operation planning should be much simpler to automate than it actually is. The basis of this difficulty is in computational complexity and the current state of computer planning algorithms. This introduction briefly explains the operational realm of joint operational planning as well as it's complexity and briefly reviews the concept of effects based planning.

"Strategic art is the essence of joint operation planning. Without mastery of strategic art, the joint operation planner cannot craft military plans that are in synergy with the strategic goals of the United States. … Lieutenant General Richard in 1996 defined strategic art as "The skillful balancing of ends (objectives), ways (courses of action), and means (resources)."" Joint Staff Officers Guide, 2000, Joint Forces Staff College Publication 1, also known as "The Purple Book". General Richard implies the key to operational planning through abstracting away the details of the plan at the strategic level. The details are required but should be abstracted away or hidden from the strategic planner. Since the inception of computer programming good programmers can generate approximately four lines of deployment capable code per day, which includes testing and documentation. This process was slow and painful when programmers worked at the machine and assembly language level of programming. As good high-level programming languages (Java, .NET or C++) have evolved the programmer still generates roughly the same four line of code per day but each line results in exponentially more machine code that gets executed by a computer processor. This low level code very rarely needs to seen by the high-level programmer. In strategic planning good abstraction is also critical for human planners to be most creative and effective in campaign planning.

Military campaign planning is likely one of the most challenging problem facing humans today because of its complexity, uncertainty and potential risk for any loss of life. For the purpose of this paper, we define planning as, determining the objectives and effects to achieve desired goal(s) through the application of military courses of action and then the decomposition of objectives into achievable sub objectives and eventually specific tasks to achieve those (sub) objectives and goals.

From a computer perspective, planning, resource allocation and scheduling are all known to be intractable, computationally very hard, "NP-complete", or stated simply not always solvable in polynomial time proportional to the size of the problem. To find the "optimal" solution to these problems the computer algorithms required contains the size of the problem in the exponent of the solution time. A problem requiring 10^n seconds to solve optimally requiring 10,000 seconds (10^4) to solve for size four, may require 10^{100} seconds for size 100. This number is longer than the theoretical age of the universe of over 6 billion years. Doubling the processing speed and use of the fastest parallel computers may only allow you to explore an additional three or four orders of size or magnitude at most. Because you can optimize a problem of four now you may only be able to optimize a problem of size eight several years latter not even coming close to problems of

hundreds and thousands you need to address.

Although good optimal approximation algorithms exist for resource allocation and scheduling problems, the planning problem (decomposition of goals to tasks to achieve that goal) is in an even harder subclass of intractable problems known as "PSPACE" (polynomial-storage space required to solve). Extensive research in the planning field i.e. ARPI[1] has yet to achieve effective planning algorithms that can generate plans as well as humans for even fairly small planning problems, likely because of this "PSPACE" nature of planning.

Understanding this complexity of automated planning and combining the current strengths of the human planners and their computers is the key to more effective planning and scheduling:

- Humans understand the context of plans, goals and objectives (often referred to as knowledge about the environment including what we call common sense). Humans also bring intuition, expertise and strategic knowledge to plan decision-making, evaluation and risk assessment.
- Computers excel at generating visualizations and tracking the details of plans like the allocation and scheduling of resources (in this case aircraft, weapons, fuel, etc.) to aid the human planner. Computers also bring quantitative methods to decision-making, evaluation and risk assessment for the human to use. [Summarized from a Mixed-Initiative briefing by James Allen of the University of Rochester]. Effectively the computer can abstract the details of the problem away from the view of the planner.

ATO Link will combine these strengths of the human planners and computers to more effectively do dynamic planning, scheduling and execution monitoring. At the start of the ARPI program in early 1990, Dr. Northrup Fowler (current AFRL/Information Directorate Chief Scientist) proposed a concept called "Interleaved Planning and Scheduling", basically allocation and scheduling of resources while the planning was occurring. As we found out in ARPI, planning was computationally to complex to entirely automate and the raw computing power required to accomplish this kind of resource allocation and scheduling was likely not available yet. In the past twelve years computing power has roughly increased more that 100 fold, making this a potential possibility now. Using the human planners to plan high level, abstract strategy and objectives, they can rapidly explore potential courses of action *(means)* to achieve objectives *(ends)* as the computers track the details (scheduling and allocation of *resources*). While exploring options the computer can visually present the quantitative results for the planner to evaluate options, assess risks and make decisions.

This approach so far looks very promising because link analysis, graph matching and mixed programming, the three technologies underlying ATO Link, have their basis in matrix mathematics, which scales very well to large problems and is computationally well behaved.

[1] (D)ARPI the Defense Advanced Research Agency & AFRL Information Directorate Planning Initiaive

Applying link analysis and graph matching as a front end helps to limit the breadth of search for potential options in the plan thus limiting the search for allocation and scheduling of resources.

21st Century Technologies Inc. did the initial proof of concept experimentation of this technique as part of a Small Business Innovative Research (SBIR) Phase I contract for the Air Force Research Laboratory's Information Systems Division's Systems Concepts and Applications Branch (AFRL/IFSA). 21st Century has been selected to continue the experimentation, scaling and development of the algorithms and concepts explored in the Phase I in a Phase II SBIR contract awarded in February 2002. C3I Associates provides subject matter expertise in crisis action and air campaign planning. The focus of this effort is primarily in the air campaign-planning domain, although we feel the techniques apply to any general planning and resource allocation problem since the problems are represented in matrices.

As a result of the September 11th attack, the ability to optimize and mine plans becomes crucial in our ability to engage in this new type of war on terrorism. The enemy we are fighting is not the cold war enemy of the Soviet Union. Nor is it a war with "rogue states" where one can at least track their military and strategic assets. In this new war on terrorism, the enemy is not a nation state but an informal group of individuals without physical assets. Therefore, *crisis action planning* technologies are particularly essential in the successful execution of this campaign against an asymmetric warfare enemy. We believe the technologies in ATO Link can also accelerate *deliberate planning*, and the resulting military plans and defense budget projections, but that is not be addressed in this small effort.

Military campaigns have to be developed and executed more quickly based on real time land, air, and space intelligence a concept referred to "Rapid Dominance" in **Joint Vision 2020**[2]. The limited resources assigned to these campaigns will have to be assigned more optimally in order to satisfy real time constraints. We are applying ATO Link using an effects-based operations (EBO) concept of operations (CONOPS) to achieve more optimal resource use. We agree with Major General (USAF) David A. Deptula's definition of EBO in his paper "**Effects-Based Operations: Change in the Nature of Warfare"**[3]. He defines effects-based operations using the term *parallel warfare*: as a construct of warfare based on achieving specific effects, not absolute destruction of target lists using simultaneous (time) application of force across each level of war uninhibited by geography (space). The focus of EBO becomes one of not destroying an opposing force using a prioritized list of targets, but compelling an ultimate purpose of a positive political outcome by using all means available political, economic, social and military means. The basic result is that instead of a traditional sequential attack first suppressing enemy air defense to gain air superiority to go after ground targets, we attack at the same time; the key leadership, essential industries, transportation connectivity with population and military forces. Although this is disputed in some EBO circles, we believe that EBO is not a

[2] http://www.dtic.mil/jv2020/jvpub2.htm
[3] http://www.aef.org/pub/papers.asp

completely new approach, we simply have never had the automated tools to fully exploit it use. C3I Associates' author Gary Illingworth will publish a history of EBO like operations shortly.

Currently fielded aerospace planning capabilities were not designed to incorporate this new EBO CONOPS. New tools will be required to assist planners in planning and determining whether the "effects-based operation" is being achieved and when it has been successful. Furthermore, tools can provide visibility into the effects-based operation decision-making during campaign planning and provide valuable insight for next generation EBO technologies.

ATO Link Technology Overview

In this paper, we outline graph matching/link analysis tools/mixed programming tools that can perform the following functionalities:

1. The ability to identify if a target, mission, package or other plan element will support the overall desired effect and/or identify if it negatively impacts the desired effects. (EBO Planning)
2. The capability for a planner to identify, for example, a set of targets that aid in obtaining the desired effect (end state). Link analysis technology shall recommend aircraft/SCL, routes, etc, that shall optimally support the desired effects without negatively impacting others.
3. The capability to determine if all the requisite resources are available to execute a plan prior to its execution. If there are currently not enough resources, a trigger can be set to warn that planner when the necessary resources are not available to achieve a plan. This is a very important capability, as currently planners may have to traverse a down long path with current tools such as the Theatre Air Planning system (TAP) only to find that there are not enough resources to execute the plan or that previous decisions have used the necessary resource where other choices were possible.
4. The ability to track the desired effect over different ATO cycles. Suppose it is determined the destruction of targets A, B, C, D causes effect E. How do we track the obtainment of this effect over different ATO periods and determine if the destruction of other targets have caused a deleterious effect? For example, during ATO_1 we might have destroyed target A and gained the effect E. At ATO_2 we might have also planned to destroy target B that was originally part of the strategy to gain effect E, however this may no longer be a requirement and the assets planned for target B could be re-planned for another target and effect. Target F may also have been destroyed with a negative effect on E. The status of reaching a desired "effect" can quickly be extracted and linked through various ATO periods. This information can also be used as a factor in optimizing plans based on determining when an effect has been realized. Thus, if targets attacked previously have appeared to support the obtainment of the desired effect E up to the current ATO period, then in the next ATO all targets associated with meeting effect E would be given higher priority. Link analysis provides a "history" as a factor in the planning and execution not just constraint-based satisfaction.
5. The ability to identify plans and schedules using purely topological processes. For example, if one considers a set of missions to be "edges" and one considers the set of "nodes" to be all the

assets that one requires to carry out a mission, then certain topological properties – such as too many edges into a single node—can be an indicator of plan deviations.

One of the goals of link analysis in the ATO context is to map effects to plans. To perform such an analysis, one requires "ground truth". Ground truths are essentially atomic propositions we know to be true or false and from which we can draw conclusions about more complex information. For our purposes, ground truth exists in Joint Targeting Toolkit/Theatre Air Planning JTT/TAP target databases. The atomic propositions we are interested in are whether or not a mission can be linked to an effect. This mapping would be implemented as a data structure where row and columns consist of effects and targets of a matrix.

Thus, if we have the relationships

- "To gain effect E, choose target T" would be represented as a "1" in the Eth row and Tth column of the matrix.
- "Target T would negatively impact effect F would be represented as a " –1" in the "Tth row" and "Fth column".
- "Target T is neutral with respect to effect G" would be represented as 0 in the "Tth row and Gth column".

Up to this point, we have only considered link analysis in the context of EBO. We now consider how link analysis can be used in the context of determining "ripple effects" in the reallocation of resources from supporting missions. Reallocation would occur in order to optimize a mission's probability of destruction (PD) rate.

With ATO 98 format one can identify how missions directly support each other by using the "support for" and "support to" sets. We shall build matrices that identify information such as mission id "alpha" supports mission id "beta. Thus, a "1" would be in the alpha and beta row and column of the matrix respectively as shown in figure 1. It could turn out that mission beta is supports mission gamma. We would traverse the path identified in the matrix to identify the next dependencies until there are no more. An entry of "0" would identify the end of a dependency.

	alpha	beta	gamma	etc	etc	etc
alpha		1	0			
beta	0		1			
gamma	0	0				
etc						
etc						
etc						

Figure 1 Mission Support Dependency Matrix. We can track mission dependencies in the matrix efficiently.

Once we identify the mission dependencies we can determine if reallocation will cause a change of effects. We can also determine if a reallocation of assets will diminish PD rates, increase

attrition, and so on. Such factors will be inserted into the optimizer that can maximize (or minimize) parameters for relevant missions. In addition, this capability can be used to develop interleaved planning and scheduling.

The last element that needs explanation is the ability to track effects in real time over different ATO cycles. This is an important capability because it provides planners a concrete and objective status into the effect during the actual air campaign. Once we know whether a mission has been successfully executed (or not) within an ATO cycle, we can enter this information into a time N targeting ATO_N matrix where a "1" means success, a "- 1" means **that action has been taken and the target has a negative effect** and 0 means action has been taken and has not yet succeeded. If the entry is empty, it means that no action has been taken. We track targets versus effects over different ATO cycles. We can easily track missions versus effects should that capability be required.

Tracking the effects over multiple ATO cycles is necessary because the desired effects may be time phased. Thus, it may be informative to the planner to see how an effect is evolving over the course of the air campaign. This can also be represented with three-dimensional matrices that track time targets and overall effects over ATO cycles. The algorithm would traverse each of the matrices by effect and retrieve overall probabilities of how well effects have been satisfied.

During the optimization process one can see how much of an effect has been satisfied over the course of the ATO cycle. Thus, one can develop "rules" such as if 50% of an effect has not been satisfied (perhaps by not yet destroying 50% of their targets), then the destruction of those targets (and the associated effect) might be given higher priority that other effects. At this stage, we identified how link analysis can be applied to Theatre Air Planning (TAP) to add effects based operations.

Using Topological Properties of Graphs for Planning and Scheduling.

Figure 2 below is an example of graph matching. The top figure is the pattern graph to be identified from the larger "input graph" at the bottom of the panel. The dark nodes in the bottom panel represent the actual matches.

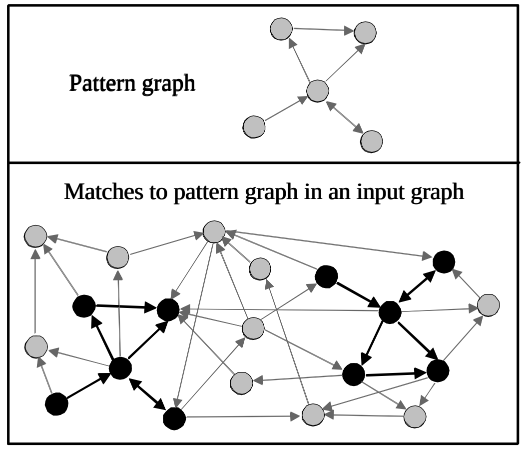

Figure 2: Example of Graph Matching

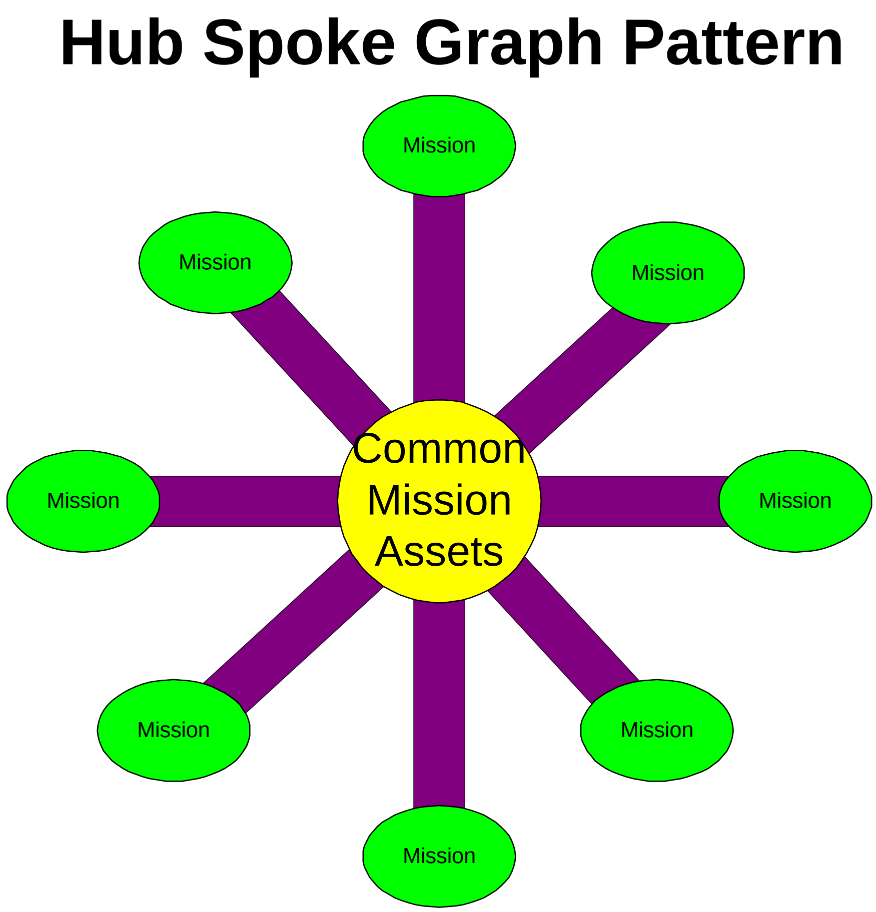

Figure 3: Hub and Spoke Pattern can represent Missions and their Common Assets.

Modeling planning and scheduling has the advantage of being able to capture certain mission dependencies. Consider a series of missions that have to be serviced by common assets within a critically short time period. This can be captured by a hub and spoke pattern that can be automatically detected using graph isomorphism algorithms developed by 21st Century

Technologies. We will explore how graph isomorphism can be applied in the area of planning and scheduling.

Interleaved Planning and Scheduling

A high risk, but potentially very high payoff will be to explore a mixed-initiative, interleaved-planning and scheduling algorithm using link analysis. Combining the planning strength of the human and resource allocation and scheduling (bean counting) strength of the computer working collaboratively generates an entire new approach to planning.

A technical approach to solving a problem algorithmically may look something like this. As the planner begins decomposing the objectives into sub-objectives and effects visually, the computer would examine potential targets previously developed by Combat Strategy intelligence personnel to satisfy these objectives and effects. The computer could use a combination of least commitment and link analysis algorithms to determine the feasibility of achieving the objectives and effects by doing detailed allocation and scheduling computations in the background with given resources and constraints. The least-commitment algorithm would not commit to specific target/ aircraft/ SCL combinations until absolutely required to do so by planner's choice of objectives/effects and constraints. If better choices are available in the current planning state visualizations being maintained by the computer can help the planner to make changes in his plan permitting more optimal use of resources. Since both planning and execution would utilize the same essential algorithms, this represents a novel approach to achieving dynamic planning and execution.

An interesting aspect of the link analysis approach is that for a deliberate planning, realistic cost figures could be used to better predict budget requirements for various theater scenarios for planning the defense budget. Using link-analysis for constraint-propagation for existing resource-constrained CONOPS is achievable but link analysis could also be used to select the best resources to deploy to achieve an Expeditionary Air Force/Effects-Based Operations CONOPS of the future.

Operational Example of Optimized Decision Making Within TAP.

To execute a direct attack on a ground target, two types of fighter squadrons (F1 and F2) require mid air refueling. There are two types of aerial tankers (T1 and T2) available for mid air refueling.

If a squadron of aerial tankers of type T1 is totally dedicated to refueling fighter type F1, then it can refuel 40 of these fighter types before having to return to base. If T1 is completely dedicated to refueling F2, then T1 squadron can refuel 60 of these fighter types before having to return to base. Similarly, the T2 aerial tanker squadron can service 50 fighter types F1, and 50 F2's before having to return to base for the day.

Both fighters are part of a package directed toward the same target. If fighter type F1 can carry only type 2 SCLs of type I and fighter type F2 can carry twice as many, four of the same SCL. **Assuming that the aerial tankers will be depleted of fuel during these missions, what is the maximum amount of SCL's that can service the target and which aircraft are optimally suited to deliver the ordnance?**

Solution:
First notice that this problem is an optimization problem. Our objective is to identify the aircraft the can deliver the maximum amount of ordnance to a target. Furthermore, we are going to maximize this number by adjusting the number of aircraft that get refueled based on fighter type. Therefore, the refueling assignments become the control/decision factors, the values of which we are called upon to determine.

In the analytical formulation of the problem, modeling them as the problem decision variables captures the role of these factors:

X_1= Number of F1's to be refueled per deployment
X_2 = Number of F2's to be refueled per deployment

In the light of the above discussion, the problem objective can be expressed analytically as:

Max $F(x_1,x_2) := 2X_1 + 4X_2$

The above equation will be called the objective function of the problem, and the coefficients 2 and 4 that multiply the decision variables in it, will be called the objective function coefficients.

Furthermore, any decision regarding the daily refueling for air missions must realize the refueling capacity of the air tankers. Hence, our next step in the problem formulation seeks to introduce these technological constraints in it. Let's focus first on the constraint that expresses the refueling capacity of the air tanker. Regarding this constraint, we know that one-day of dedicated refueling work for T1 can result in 40 refuelings of a fighter type F1, while the same period dedicated to 60 refuelings of fighter F2. Assuming that refueling of each fighter is a constant amount of time, then refueling 1 fighter F1 can take $p_{11} = 1/60$ of available fuel and refueling fighter F2 can take $p_{12} = 1/40$ of available fuel. The total capacity required for refueling per deployment of X1 by T1 and X2 by T1 is equal to $1/50\, X_1 + 1/50\, X_2$.. Hence, the technological constraint imposing the condition that our total refueling capability should not exceed its capacity, is analytically expressed by:

$$1/40\, X_1 + 1/60\, X_2 \leq 1$$

In this case, the right hand side of the equation represents the total amount of refuelings per deployment. Following the same reasoning, the additional constraint representing the finite refueling capacity of tanker T2 is represented by

$$\frac{1}{50} X_1 + \frac{1}{50} X_2 \leq 1$$

Finally, to the above constraints we must add the requirement that any permissible value for variables X_1, X_2 must be nonnegative, i.e. $Xj > 0$ for $j = 1,2..$, since these values express number of refuelings. These constraints are known as the variable sign restrictions.

Combining these equations, our analytical formulation is now:

Max $F(x_1,x_2) := 2 X_1 + 4 X_2$

such that

$$\frac{1}{40} \mathbf{X_1} + \frac{1}{60} \mathbf{X_2} \leq 1$$

$$\frac{1}{50} \mathbf{X_1} + \frac{1}{50} \mathbf{X_2} \leq 1$$

$X_j > 0$ for $j = 1,2..$

At this stage, we have set up the equations necessary to compute a solution. We do now describe below what the solution space should consist of.

- The Linear Programming Solution Space for Inequality Constraints

A single inequality constraint is of the form

$$aX_1 + bX_2 \leq C$$

The solution space for the above inequalities is identified in the shaded area of figure XX.

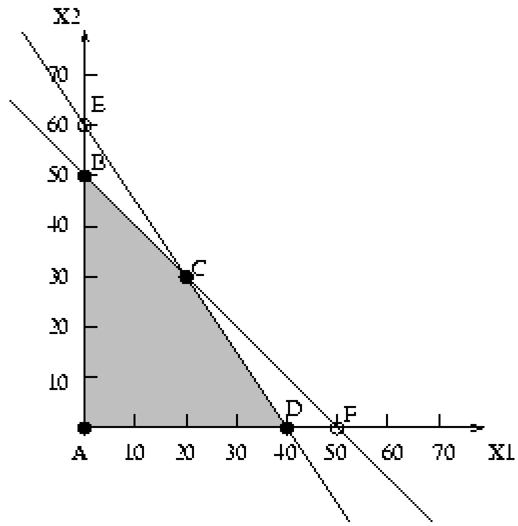

Figure 2 Solution Space of the types of constraint as shown above (single inequality constraint) is represented as half plane.

- Representing the Objective Function in LP solution space

In linear programming, the objective function will always be a straight line. The optimal solution for our scenario is represented in figure 2. In the linear programming case and in our example, the optimal solution will be one of the points "B", C", "D", "E" and so on shown in the above figure. From this figure, it follows that the optimal daily values are given by the coordinates of the point corresponding to the intersection of line $\frac{1}{50}X_1 + \frac{1}{50}X_2 = 0$ with the X2-axis, i.e., $X^{opt}_1 = 0$ and $X^{opt}_2 = 50$. This is point B. Therefore, the maximum number of SCL Is that can be delivered is 200. Max $F(x_1,x_2) := 2 * 0 + 4 * 50 = 200$. The optimal point is point B as shown in the figure 3. We refer to this point as the "extreme" point.

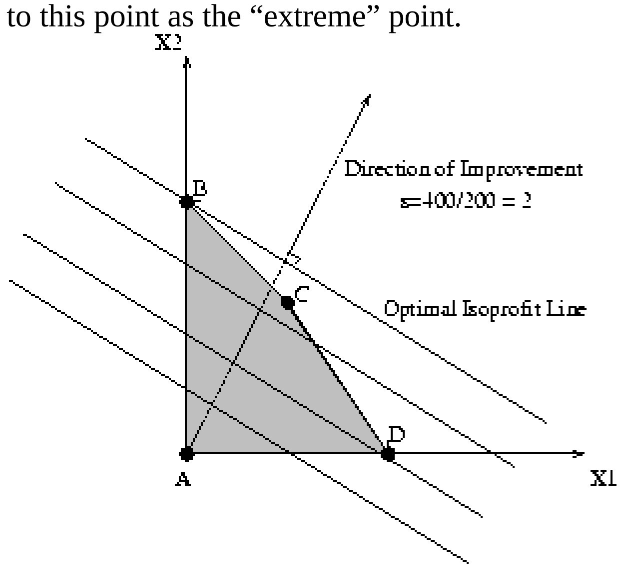

Figure 3 Optimal Objective Function Line is line intersecting point "B".

The SIMPLEX METHOD

One of the classic methods for determining optimality in the linear case is the simplex method. To describe the solution, we need to modify our set of equations into "standard" form.

Max $F(x_1,x_2) := 2X_1 + 4X_2$

such that

$$\tfrac{1}{40} X_1 + \tfrac{1}{60} X_2 + S_1 = 1$$

$$\tfrac{1}{50} X_1 + \tfrac{1}{50} X_2 + S_2 = 1$$

$X_j, S_k > 0$ **for** $k,j = 1,2..$

This formulation involves four variables and two technological constraints. By definition, a "basic" solution will be defined by selecting a basis of two variables, with the remaining two being set equal to zero. To find point B which is the optimal solution of this example, set $\{X_2, S_1\}$ as the basis and therefore $X_1 = S_2 = 0$. We can compute the values for these variables by solving the system of equations to return point B.

$$\tfrac{1}{60} X_2 + S_1 = 1$$

$$\tfrac{1}{50} X_2 = 1$$

One might think that a (naive) approach to the problem would be to enumerate the entire set of extreme points, compare their corresponding objective values, and eventually select one which minimizes the objective function over this set. For example, a small LP with 10 variables (in "standard form") and 3 technological constraints can have up to 120 extreme points, while an LP with 100 variables and 20 constraints can have up to 2000 extreme points.

Hence, we need a more systematic approach to organize the search so that we manage the complexity resulting from the size of the search space. The Simplex algorithm provides such a systematic approach. The basic logic of the algorithm is depicted in the figure 4 below.

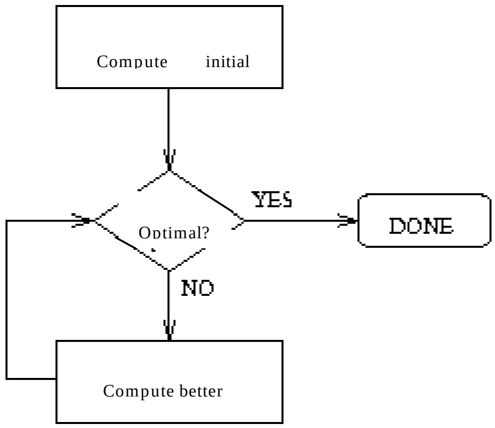

Figure 4 Simplex Algorithm Logic

The algorithm starts with an initial basic solution and tests its optimality. If some optimality condition is verified, then the algorithm terminates. Otherwise, the algorithm identifies an adjacent basic solution, with a better objective value. The optimality of this new solution is tested again, and the entire scheme is repeated, until an optimal basic solution is found. Since every time a new basic solution is identified the objective value is improved and the set of basic solutions is finite, it follows that the algorithm will terminate in a finite number of steps (iterations).

It is also interesting to examine the geometrical interpretation of the behavior of Simplex algorithm. Given the above description of the algorithm and the correspondence of basic solutions to extreme points, it follows that Simplex essentially starts from some initial extreme point, and follows a path along the edges of the feasible region towards an optimal extreme point, such that all the intermediate extreme points visited are improving (more accurately, not worsening) the objective function.

The simplex method is one of many solutions available in linear programming. We shall investigate a number of open source software solutions.

The simplex in the simplex algorithm is the collection of vertices that support the convex region that represents the feasible set of solutions. These vertices are connected along edges that along with the vertices are at the boundary of the feasible set. The simplex algorithm is the traversal from one vertex to another along connected vertices in such a manner that the objective function increases during the traversal. The algorithm terminates when all edges from a point results in reduced objective function. The collection of vertices constitutes the set from which the solution is found.

Other Optimization Algorithms

The Simplex algorithm is the starting point of a variety of algebra based optimization algorithms. The other class of optimization algorithms is based on calculus. We will discuss some of those later.

Integer programming: Integer programming differs from linear programming in that the decision space is confined to integers for some of the variables.
We need to solve integer programs because we may have (as we do in many plans) a case where we have to make choices about what things to take. For example we may be choosing one from a variety of SCL's or we may be taking several. So we cannot talk about a fractional SCL. We take an integer of SCLs or nothing at all. The plan then needs to tell us how many we will take.
One may be tempted to solve this problem by assuming that the variables are continuous and find the nearest integer solution. However there is no guarantee that an optimal integer solution in any neighborhood of the optimal continuous solution. It is very easy to generate integer programming problems where the solution obtained by rounding the linear programming solution is infeasible while the optimal solution is as far as we like either in the value of the maximized cost function or in terms of distance in solution space.
The branch and bound method is used with the general linear programming solution to solve integer-programming problems. It is essentially a divide and conquer routine where the feasible region is partitioned into more manageable sub-regions. In addition, any feasible integer solution represents a lower bound on the objective function, allowing us to avoid linear programming computations in regions where we already have obtained a better solution. The solution of the continuous solution represents an upper bound on the feasible region. The regions are divided along integer boundaries that surround the continuous solution until either no feasible region exists or a sub-optimal integer solution is found in some subdivision. All of the solutions along the generated tree are compared to get the optimal answer.
The integer programs are known to be NP-Complete. While this would give one pause when attempting to solve these problems, these kinds of problems are solved on a regular basis and should not be a hindrance to the use of integer programming solutions. In general however even with machines that are far beyond what anyone could have sitting on their desk a couple of years ago, it may be necessary to abandon the goal of a provable optimum; by terminating a integer programming code prematurely, you can often obtain a high-quality solution along with a provable upper bound on its distance from optimality. A solution for the objective value within some fraction of 1% of optimal may be all that is required. In contrast to methods for ordinary linear programming, procedures for integer programming may not be able to prove a solution to be optimal until long after the optimal solution has been found.
There are concepts that cannot be expressed well in a linear program. For example we may need to compare two of one asset compared to two different assets. Its hard to make this comparison in a linear program but easy to make this comparison in a non linear program. In particular quadratic programming allows a more sophisticated optimization criteria given the same constraints as the linear program.

Conclusion:

This paper presents an innovative approach to military resource planning, allocation, and scheduling based on link analysis and mixed programming (mixed integer linear programming

problem). ATO Link (Air Tasking Order - Link Analysis) will link and more optimally identify the resources through out the aerospace campaign planning and execution phases using an effects-based operations (EBO) concept of operations and approach. This approach so far looks very promising because link analysis and mixed programming, the two technologies underlying ATO Link, have their basis in matrix mathematics, which scales very well to large problems and is computationally fairly well behaved. Applying link analysis as a front end helps to limit the breadth of search for potential options in the plan thus limiting the search for use of resources. A high risk, but potentially very high payoff will be to explore a mixed-initiative, interleaved-planning and scheduling algorithm using link analysis. Combining the planning strength of the human and resource allocation and scheduling (bean counting) strength of the computer working collaboratively generates an entire new approach to planning.